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An Analysis of Speeding Multicast by Acknowledgment Reduction Technique (SMART) with Homogeneous and Heterogeneous Links - A Method of Types Approach†

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I. Introduction

In current data networks there are many applications that support multicast from a single source to multiple receivers, such as conferencing, online gaming, and video streaming. Among these are those applications that require *reliable* mutlicast, such as reliable bulk data transfer applications that impose strict reliability and a received file is considered invalid even if one bit is received incorrectly.

Reliability in multicast networks is achieved through Forward Error Correction(FEC), Automatic Repeat Requests(ARQ), and other feedback mechanisms. SMART [1] is a novel NAK-based feedback protocol, that asymptotically reduces the total transmission time of a file transfer to that of an omniscient transmitter that knows the state of every receiver and link at all times. SMART provides a mechanism for a potentially large set of receivers to send their feedback in a *single* time slot and uses a predictive model to choose the optimal feedback time. This combination allows a significant reduction of unnecessary feedback thus resulting in shorter total transmission times.

In this paper we apply the information theoretic concept of *Method of Types* to characterize optimal throughput of the erasure broadcast channel. This concept is then utilized to develop a computationally efficient expression that gives the optimal feedback time in a multicast setting whose transmitter uses network coding. We then generalize this approach to address a multicast network with heterogeneous links of different rate, and different packet erasure probability. This method can greatly help in speeding the delivery time over heterogeneous networks.

The remainder of the paper is organized as follows: In Section II, we explain Method of Types and the framework through which it can be applied to various setups. Section III, discusses a broadcast channel with homogeneous links. Section IV, extends the analysis to a network with two heterogeneous links. Section V, generalizes our approach to a broadcast network with many heterogeneous links. Finally, in Section VI we compare the performance of SMART to two other transmission strategies and show its robust performance.

II. General Theory and Method of Types

Consider an $n \times t$ matrix $\mathcal{X}$ whose $(i,j)^{th}$ element, $X_{i,j}$, is a Bernoulli random variable with parameter P, where $P = Pr\{X_{i,j} = 1, \forall(i,j)\}$. Let us denote the i^{th} row of $\mathcal{X}$ by the vector $[\mathbf{X}^t]_i$ which is a sequence of t i.i.d. Bernoulli random variables with the same parameter P. In other words, $[\mathbf{X}^t]_i = X_{i,1}X_{i,2}...X_{i,t}$. We associate a random variable $S_i(t)$ with the i^{th} row of this matrix that represents the number of 1's in that row. The notation $S_i(t)$ emphasizes the dependence of this random variable on the length t of each sequence. Thus, $S_i(t) = \sum_{j=1}^{t} X_{i,j}$.

We are interested in statistical behavior of the collection of all $[\mathbf{X}^t]_i \in \mathcal{X}$ and will use the notion of *Types* introduced by [2] to classify them. A given sequence $[\mathbf{X}^t]_i$ is said to be of type Q, if it has an empirical distribution $\frac{S_i(t)}{t} = Q$. Note that there are exactly $t+1$ possible types, each corresponding to a different ratio of 1's in the sequence. Let us calculate the probability

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of $[\mathbf{X}^t]_i$ with type Q:

$$\begin{aligned}
Pr\left\{[\mathbf{X}^t]_i : \frac{S_i(t)}{t} = Q\right\} &= Pr\left\{X_{i,j}=1\right\}^{S_i(t)} Pr\left\{X_{i,j}=0\right\}^{t-S_i(t)} \\
&= P^{tQ}(1-P)^{t(1-Q)} && (1)\\
&= \exp\left[t\left(Q\ln(P)+(1-Q)\ln(1-P)\right)\right] && (2)\\
&= \exp\left[t\left(Q\ln(Q)+(1-Q)\ln(1-Q)+Q\ln\left(\frac{P}{Q}\right)+(1-Q)\ln\left(\frac{1-P}{1-Q}\right)\right)\right] \\
&= \exp\left[-t\left(H_b(Q)+D\left(Q\|P\right)\right)\right] && (3)
\end{aligned}$$

where $H_b(\cdot)$ represents the binary entropy function measured in nats, and $D(\cdot\|\cdot)$ is the Kullback-Leibler (KL) divergence between two Bernoulli distributions. The following symmetries hold for $H_b(\cdot)$ and $D(\cdot\|\cdot)$ if the input distributions are binary:

$$\begin{aligned}
H_b(Q) &= H_b(1-Q) && (4)\\
D\left(Q\|P\right) &= D\left((1-Q)\|(1-P)\right) && (5)
\end{aligned}$$

Given that (3) can efficiently calculate the probability of a sequence with a particular empirical distribution, we use it to calculate the probability that certain empirical distributions are present/missing in $\mathcal{X}$. Let us define two new random variables $S_{min}(t,n,P) = \min_i S_i(t)$ and $S_{max}(t,n,P) = \max_i S_i(t)$ to denote minimum/maximum number of 1's among the rows of $\mathcal{X}$. Our initial functions of interest are the distributions of S_{min} and S_{max}. Let $\beta_{min}(t,n,i,P)$ be the probability that the minimum number of 1's is greater or equal to i for all sequences in $\mathcal{X}$. Thus:

$$\begin{aligned}
\beta_{min}(t,n,i,P) &= Pr\left\{S_{min}(t,n,P) \geq i|\mathcal{X}\right\} && (6)\\
&= \left[Pr\left\{[\mathbf{X}^t]_i \in \mathcal{X}, Q \in \left[\frac{i}{t},1\right]\right\}\right]^n && (7)\\
&= \left[\sum_{j=i}^{t}\binom{t}{j}\exp\left[-t\left(H_b\left(\frac{j}{t}\right)+D\left(\frac{j}{t}\|P\right)\right)\right]\right]^n && (8)
\end{aligned}$$

Similarly, let $\beta_{max}(t,n,i,P)$ be the probability that the maximum number of 1's is less than or equal to i for all sequences in $\mathcal{X}$:

$$\begin{aligned}
\beta_{max}(t,n,i,P) &= Pr\left\{S_{max}(t,n,P) \leq i|\mathcal{X}\right\} && (9)\\
&= \left[Pr\left\{[\mathbf{X}^t]_i \in \mathcal{X}, Q \in \left[0,\frac{i}{t}\right]\right\}\right]^n && (10)\\
&= \left[\sum_{j=0}^{i}\binom{t}{j}\exp\left(-t\left(H_b\left(\frac{j}{t}\right)+D\left(\frac{j}{t}\|P\right)\right)\right)\right]^n && (11)
\end{aligned}$$

Note that β_{min} and β_{max} are cumulative probabilities and we can get the Probability Mass Functions (pmf) of S_{min} and S_{max} from them. Let $\alpha_{min}(t,n,i,P)$ be the probability that $S_{min}(t,n,P) = i$ then:

$$\alpha_{min}(t,n,i,P) = \beta_{min}(t,n,i,P) - \beta_{min}(t,n,(i+1),P) \tag{12}$$

Similarly, let $\alpha_{max}(t,n,i,P)$ be the probability that $S_{max}(t,n,P) = i$, then:

$$\alpha_{max}(t,n,i,P) = \beta_{max}(t,n,i,P) - \beta_{max}(t,n,(i-1),P) \tag{13}$$

Notice that:

$$\begin{aligned}
\beta_{min}(t,n,0,P) &= 1 \qquad \beta_{min}(t,n,t,P) = \alpha_{min}(t,n,t,P) = (1-P)^{nt} && (14)\\
\beta_{max}(t,n,t,P) &= 1 \qquad \beta_{max}(t,n,0,P) = \alpha_{max}(t,n,0,P) = (P)^{nt} && (15)
\end{aligned}$$

$$\begin{aligned}
\beta_{min}(t,n,j,P) &= 0 \qquad \forall j > t && (16)\\
\beta_{max}(t,n,j,P) &= 0 \qquad \forall j < 0 && (17)
\end{aligned}$$

As a result we can generate the desired pmfs by first calculating the β functions and then using the the recursive equations (12) and (13) to get the pmfs.

Most importantly we are interested in calculating the expected value of S_{min} and S_{max} whose pmfs for a fixed t are (12) and (13) respectively. In other words, we want to know the expected minimum/maximum number of ones among the rows of $\mathcal{X}$, and as we will show shortly we do not need the α functions to make this calculation.

For a non-negative discrete random variable X, we can find the expected value as:

$$E[X] = \int_0^\infty Pr\{X > x\}\,dx = \int_0^\infty Pr\{X \geq x\}\,dx$$

where the last equality holds because $Pr\{X \geq x\}$ and $Pr\{X > x\}$ are the same except at a finite set of discontinuities, and does not affect the integral. Since S_{min} and S_{max} are non-negative, we have:

$$E[S_{min}(t,n,P)] = \int_0^\infty \beta_{min}(t,n,\theta,P)d\theta \quad (18)$$

$$= \sum_{i=1}^{t} \beta_{min}(t,n,i,P) \quad (19)$$

$$E[S_{max}(t,n,P)] = \int_0^\infty (1-\beta_{max}(t,n,\theta,P))\,d\theta \quad (20)$$

$$= \sum_{i=1}^{t} (1-\beta_{max}(t,n,i,P)) \quad (21)$$

where we have used (16) and (17) to get (19) and (21).

Another parameter of interest is the distribution of the required length of the sequences, t, that ensures a minimum number of ones in each of the n rows of $\mathcal{X}$. In other words, given that we require a minimum of k ones in each of the n sequences, what is the probability that this constraint is met with strings of length t. To state this more carefully, note that we are interested in the probability that the minimum number of ones among the n sequences of length t is greater than or equal to k (rather than only equal to). As an example, consider the difference $\beta_{min}(t+1,n,k,P) - \beta_{min}(t,n,k,P)$. This difference shows the probabilistic gain of increasing the length of each sequence by one more bit. After some thought it becomes clear that for a fixed (n,k,P), the probability of interest is exactly $\beta_{min}(t,n,k,P)$. Using the non-negativity of t we can find the expected (minimum) required length, $E[t_{min}]$, for a given set of constraints (n, k,P):

$$E[t_{min} \mid (n,k,P)] = \int_0^\infty (1-\beta_{min}(\theta,n,k,P))\,d\theta$$

$$= k + \sum_{i=k}^{\infty} 1 - \beta_{min}(i,n,k,P) \quad (22)$$

III. Networks with Homogeneous Links

Having developed the necessary tools, let us revisit the problem of transmitting k packets to a set $\mathcal{N}$ of receivers[1] over a broadcast erasure channel with erasure probability p_1. Assume that erasures occur independently across time and users, and let us use 0s to denote erasures and 1s to represent successful receptions at a receiver. This representation allows us to record the complete outcome of the transmissions up to and including time t in an $n \times t$ matrix $\mathcal{X}$ where the i^{th} row $[\mathbf{X}^t]_i$ represents the transmission outcomes at the i^{th} receiver. Let us define $\gamma(t)$ to be the probability that every receiver has finished the download by time t, this is the probability that the minimum number of 1's in each row of $\mathcal{X}$ is greater than or equal to k. From (8) we have:

$$\gamma(t) = Pr\{S_{min} \geq k|\mathcal{X}\} = \beta_{min}(t,n,k,(1-p_1)) \quad (23)$$

Fig. 1 depicts $\gamma(t)$ for a range of erasure probabilities. Notice that $\gamma(t)$ undergoes a sharp phase transition especially for smaller erasure probabilities. Our goal is to choose the time of the initial feedback when $\gamma(t)$ is sufficiently large such that the total download time is minimized. Let t_1 be the time of the initial feedback, and $t_2(t_1)$ be the number of retransmissions needed (after t_1) to complete the download. Notice that the number of retransmissions is strongly dependent on the time of the initial feedback justifying the notation.

Note that with SMART, we can obtain the feedback from all receivers in a *single* time slot. Thus, the total transmission time T_{tot} can be written as $T_{tot} = (t_1+1)+(t_2(t_1)+1)$, where the two ones account for the number of slots allocated for feedback

[1]We will use n and n_1 to refer to the cardinality of the sets $\mathcal{N}$ and $\mathcal{N}_1$

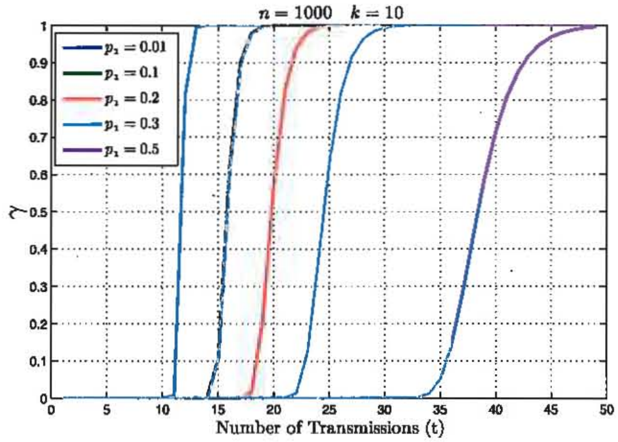


Figure 1: Completion probability as a function of t for different values of p_1.

in each round. We are interested in choosing t_1 to minimize the expected total transmission time. Note that $(1-\gamma(t_1))$ is the probability that retransmissions are needed after the initial feedback and:

$$\begin{aligned} E\left[T_{tot}|t_1\right] &= (t_1+1)+(1-\gamma(t_1))\left(E\left[t_2(t_1)|\ t_1\right]+1\right) &(24)\\ t_1 &= \underset{t}{\operatorname{argmin}}\ E\left[T_{tot}|t\right] &(25)\end{aligned}$$

We can accurately bound $E\left[t_2(t_1)|t_1\right]$ by considering two simple scenarios that will follow shortly. Recall that S_{min} denotes the minimum number of 1's among the rows of $\mathcal{X}$, in other words S_{min} is the feedback received from the receiver that has encountered the highest number of erasures. Given that at t_1, $S_{min}=i$ the transmitter will use a predetermined strategy to calculate and transmit $t_2(t_1)$ packets to complete the download:

$$\begin{aligned} E\left[t_2(t_1)|\ t_1\right] &= \sum_{i=0}^{k-1} Pr\left(S_{min}=i|t_1\right) E\left[t_2(t_1)|\ t_1, S_{min}=i\right] \\ &= \sum_{i=0}^{k-1} \alpha_{min}(t_1,n,i,(1-p_1))\ E\left[t_2(t_1)|\ t_1, S_{min}=i\right] &(26)\end{aligned}$$

where the limits of the sum are chosen to only consider the cases where $t_2(t_1)$ is non-zero. We can lower bound (26) by assuming that there is only 1 node that has received this minimum number of packets, and upper bound it by assuming that all n receivers have received the same minimum number of packets, using (22) we have:

$$\begin{aligned} E\left[t_2(t_1)|\ t_1, S_{min}=i\right] &\geq E\left[t_{min}|1,i,(1-p_1)\right] \\ &= \frac{i}{1-p_1} &(27)\\ E\left[t_2(t_1)|\ t_1, S_{min}=i\right] &\leq E\left[t_{min}|n,i,(1-p_1)\right] &(28)\end{aligned}$$

The initial work on SMART [1], suggested a scheme in which the transmitter scales up the number of packets requested by the worst receiver by a factor of $(1-p_1)$ which is simply the lower bound shown above. A tighter lower bound can be expressed as:

$$E\left[t_2(t_1)|\ t_1, S_{min}=i\right] \geq \sum_{j=1}^{n} Pr\left\{\begin{array}{c}\text{that exactly } j \text{ nodes}\\ \text{have missed } i \text{ packets}\end{array}\right\} E\left[t_{min}|j,i,(1-p_1)\right]$$

IV. Networks with Two Heterogeneous Links

In this section, we consider the transmission of a file of k packets from a single source to a set $\mathcal{N}$ of receivers. The source transmits over two links of rates R_1 and R_2 packets per unit time. The links have packet erasure probabilities p_1 and p_2 respectively. We assume that a subset $\mathcal{N}_1$ of the receiving nodes can only receive over the first link and the remaining nodes in $\overline{\mathcal{N}_1}$ can receive over both links. The goal of the transmitter is to send a file of k packets to all receivers in the shortest possible time using SMART. Such network topologies arise frequently when a set of mobile devices with access to multiple links move through a region with non-uniform coverage. As an example, consider current generation of smart phones that can simultaneously communicate over Wi-Fi and 3G, each of which has a different rate and packet erasure probability. A multicast session with such devices may include phones whose Wi-Fi or 3G connection is disrupted, leaving them with only one communication channel.

We propose a scheme where the base station transmits *independent* coded packets on each link and as a result any successful reception will provide a new degree of freedom at the receivers. This coding strategy transforms the problem to a scenario where the base station transmits packets at a rate R_1 to n_1 nodes, while transmitting at a rate of $R_1 + R_2$ to $n - n_1$ nodes, and as before we are interested in finding a feedback time to minimize the expected total download time.

Let us consider the nodes that have access to both links. The successful transmissions to node $i \in \overline{\mathcal{N}_1}$ can be modeled as a binary sequence $\left[\mathbf{X}^{t(R_1+R_2)}\right]_i$. This sequence represent a Bernoulli arrival process formed by merging[2] two independent Bernoulli processes $\left[\mathbf{X}^{tR_1}\right]_i$ and $\left[\mathbf{X}^{tR_2}\right]_i$, corresponding to the transmissions on different links. We can define a new parameter p_3 to denote the probability that the j^{th} element of the combined sequence is a 0:

$$\begin{aligned} p_3 &= Pr\left\{X_{i,j} = 0 | i \in \overline{\mathcal{N}_1}\right\} \\ &= Pr\left\{X_{i,j} \in \left[\mathbf{X}^{tR_1}\right]_i\right\} Pr\left\{X_{i,j} = 0 | i \in \overline{\mathcal{N}_1}, X_{i,j} \in \left[\mathbf{X}^{tR_1}\right]_i\right\} \\ &+ Pr\left\{X_{i,j} \in \left[\mathbf{X}^{tR_2}\right]_i\right\} Pr\left\{X_{i,j} = 0 | i \in \overline{\mathcal{N}_1}, X_{i,j} \in \left[\mathbf{X}^{tR_2}\right]_i\right\} \\ &= \frac{R_1 p_1 + R_2 p_2}{R_1 + R_2} \end{aligned} \tag{29}$$

The expressions above suggest that we can treat the nodes that have access to both links similarly to the nodes in $\mathcal{N}_1$ if we define a new rate $R_3 = R_1 + R_2$ with packet erasure probability p_3 as defined in (29). Using binary matrices to represent the outcome of transmissions, we notice that because of the variable rates experienced by different users we have to use differently sized matrices. Let $\mathcal{X}_{\mathcal{N}_1}$ and $\mathcal{X}_{\overline{\mathcal{N}_1}}$ denote the transmission matrices for the nodes in $\mathcal{N}_1$, and the nodes in it's complement $\overline{\mathcal{N}_1}$ respectively.

As before, let us use $\gamma(t)$ to denote the probability that every receiver has completed the download by time t. Recall that for each transmission matrix we can calculate $S_{min} = \min_i S_i(t)$, and download is completed if S_{min} is greater that or equal to k for both matrices. Thus we have:

$$\begin{aligned} \gamma(t) &= Pr\left\{S_{min} \geq k | \mathcal{X}_{\mathcal{N}_1}\right\} Pr\left\{S_{min} \geq k | \mathcal{X}_{\overline{\mathcal{N}_1}}\right\} \\ &= \beta_{min}\left(tR_1, n_1, k, (1-p_1)\right) \beta_{min}\left(tR_3, (n-n_1), k, (1-p_3)\right) \end{aligned} \tag{30}$$

Following the rational of Section III and using above definition of $\gamma(t)$, we can get the optimal feedback time t_1:

$$E\left[T_{tot}|t_1\right] = (t_1+1) + (1-\gamma(t_1))\left(E\left[t_2(t_1)|\ t_1\right]+1\right) \tag{31}$$

$$t_1 = \underset{t}{\operatorname{argmin}}\ E\left[T_{tot}|t\right] \tag{32}$$

Let us discuss the behavior of $\gamma(t)$ in (30) before considering any bounds for $E\left[t_2(t_1)|\ t_1\right]$. Recall that β_{min} is a CDF that transitions sharply from 0 to 1. The probability that every node has completed the download by time t, $\gamma(t)$, is the product of two such β_{min} functions and will resemble the β_{min} that has a delayed phase transition period. An extreme example of this notion is the product of two step functions and we can clearly see that the product is equal to the delayed step function. In other words, the download completion time is most severely affected by the set of users whose β_{min} function is delayed. If we use this "worst" β_{min} function as an approximation to $\gamma(t)$ we can use equations (26-28) to find the expected completion time. Fig. 2 illustrates the accuracy of this approximation.

The exact expression for the expected minimum number of dofs received via two heterogeneous links by time t is shown below and can be used to estimate the total transmission time.

$$\begin{aligned} E\left[S_{min}(t,n,P)\right] &= \int_0^{\infty} \beta_{min}(tR_1, n_1, \theta, (1-p_1))\beta_{min}(tR_3, (n-n_1), \theta, (1-p_3))d\theta \\ &= \sum_{i=1}^{t} \beta_{min}(tR_1, n_1, i, (1-p_1))\beta_{min}(tR_3, (n-n_1), i, (1-p_3)) \end{aligned} \tag{33}$$

The plots in Fig. 3, show that even a single node in $\mathcal{N}_1$ drastically affects the total transmission time. Also notice that the erasure probability of the second link does not affect the overall performance of the system.

[2]The merging that we are discussing in this section is slightly different from the familiar Bernoulli merging processes taught in elementary courses of stochastic processes. Traditionally, we consider an arrival in the merged process if there was an arrival in at least one of the original processes. In other words, if there is an arrival at time i of both incoming processes we would only register one of them in the merged process and the number of arrivals was not conserved. In our current definition we avoid this problem by assigning the would-be discarded arrival at random among time indices that do not have an arrival.

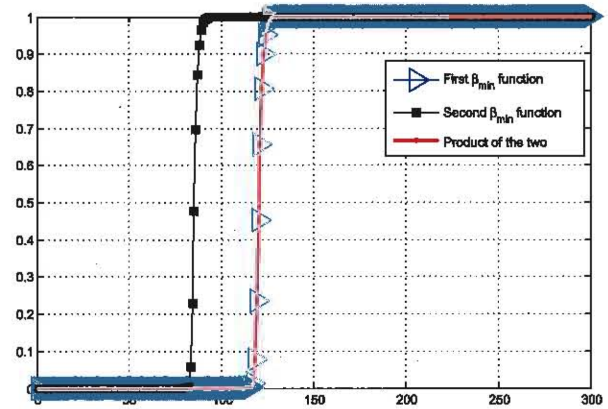
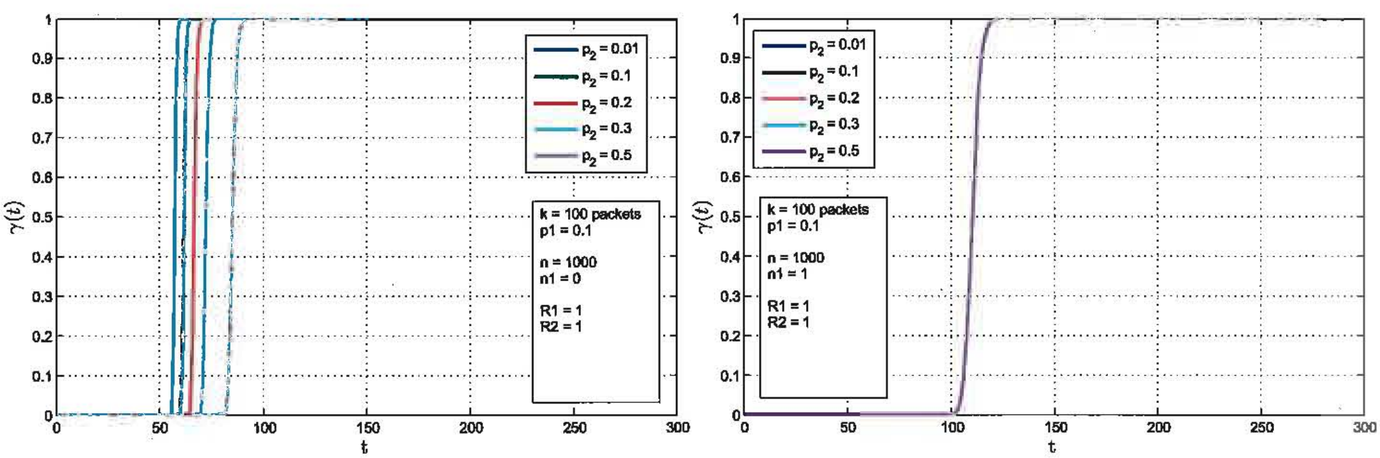


Figure 2: Depiction of $\gamma(t)$ as the product of two β_{min} functions



(a) (b)

Figure 3: Depiction of $\gamma(t)$ for $n_1 = 0$ and $n_1 = 1$ for a range of p_2

V. General Single-Hop Networks

Consider a broadcast network with a transmitter and a set $\mathcal{N}$ of receivers, the transmitter has a file of k packets to transmit to everyone. If the receivers can be classified into M subgroups, $\{\mathcal{N}_1, \mathcal{N}_2, ..., \mathcal{N}_M\}$, based on their ability to access different links, we can define a new packet erasure probability $\{p_1, p_2, ..., p_M\}$ for each group using (29), and an independent rate $\{R_1, R_2, ..., R_M\}$. We can then use the notions developed in the previous section to write out the expression for the $\gamma(t)$, the probability that every node has received every packet at time t:

$$\gamma(t) = \prod_{i=1}^{M} \beta_{min}(tR_i, n_i, k, (1-p_i)) \tag{34}$$

As before, we can approximate $\gamma(t)$ by the β_{min} function that is delayed the most, and other arguments will follow.

VI. Comparison to other transmission strategies

In this section we discuss two other transmission strategies that are often employed in heterogeneous networks but are inferior to SMART as was discussed here.

Consider the heterogeneous network discussed in Section IV with $R_1 = R_2 = 1$ and $n_1 = 0$. These parameters express a scenario where a transmitter uses two independent links of the same rate (but different packet erasure probability) to transmit a file of k packets to n nodes. A somewhat rational strategy is to divide the file according to the throughput of each link and transmit each portion independently. In other words, the first link will be assigned $\frac{1-p_1}{2-(p_1+p_2)}k$ packets and the second link will have the remaining $\frac{1-p_2}{2-(p_1+p_2)}k$ packets, and each link will only code across the packets assigned to it. As a result, a given receiver will complete the download if and only if both portions are received in their entirety. Let $\gamma_{1^{st}St.}(t)$ be the probability that every node has completed the download by time t:

$$\gamma_{1^{st}St.}(t) = \beta_{min}\left(t, n, \left(\frac{1-p_1}{2-(p_1+p_2)}\right)k, (1-p_1)\right)\beta_{min}\left(t, n, \left(\frac{1-p_2}{2-(p_1+p_2)}\right)k, (1-p_2)\right) \tag{35}$$

The second strategy codes across the entire file but transmits the same coded packet on both links. In this case, a receiver obtains a new degree of freedom if it receives at least one of the two packets transmitted at that time. In other words, the strategy has effectively reduced the packet erasure probability to $p_1 p_2$ but the transmission rate has not changed. As a result, the completion probability for this strategy is:

$$\gamma_{2^{nd}St.}(t) \quad = \quad \beta_{min}(t, n, k, (1 - p_1 p_2)) \tag{36}$$

If we use SMART, whereby we code across the entire file and send independently coded packets across each link, a receiver will complete the download if it has successfully received k or more dofs from either of the links. Thus:

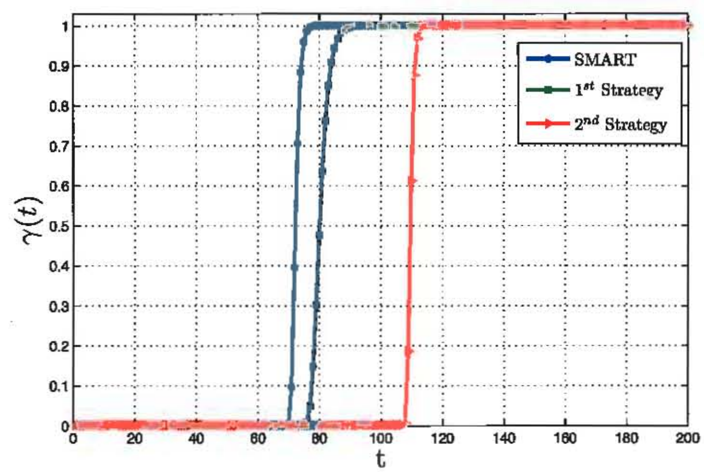
$$\gamma_{SMART}(t) \quad = \quad \beta_{min}\left(2t, n, k, \left(1 - \frac{p_1 + p_2}{2}\right)\right) \tag{37}$$

Recall that $\gamma(t)$ is a measure of how close we are to completing the download. It can be shown that:

$$\gamma_{1^{st}St.}(t) \quad < \quad \gamma_{SMART}(t) \qquad \forall t > k \tag{38}$$

$$\gamma_{2^{nd}St.}(t) \quad < \quad \gamma_{SMART}(t) \qquad \forall t > k \tag{39}$$

proving the superior performance of SMART. The following figure plots the $\gamma(t)$ for all three strategies for a network with of $n = 1000$ nodes with a file of $k = 100$ packets, and packet erasure probabilities $(p_1, p_2) = (0.1, 0.3)$. The plot confirms two well known criteria for transmission across heterogeneous networks, first is to code across the entire file and second to avoid correlation at the transmitters.



(a)

Figure 4: Depiction of $\gamma(t)$ for different strategies

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